Int. J. Heat Mass Transfer. Vol. 36, No. 14, pp. 3411–3419, 1993
Printed in Great Britain

0017-9310/93 $6.00+0.00

Study on liquid laminar free convection with consideration of variable thermophysical properties

DE-YI SHANG,† BU-XUAN WANG,‡ YANG WANG† and YUI QUAN§

† Department of Ferrous Metallurgy, Northeastern University, Shenyang 110006, China
‡ Thermal Engineering Dept., Tsinghua University, Beijing 100084, China
§ Zhengzhou Aluminium Works, Henan (Province) 450000, China

(*Received* 21 *July* 1992 *and in final form* 20 *February* 1993)

Abstract—The analyses focus on liquid laminar free convection along an isothermal vertical flat plate with consideration of variable fluid thermophysical properties. Measurements on velocity fields were made for water laminar free convection with different plate-surface temperature conditions by LDV (laser doppler velocimeter). The measured results agree very well with the calculated velocity profiles. Expressions for local Nusselt number, $Nu_{x,\infty}$, depending on the temperature gradient $d\theta/d\eta|_{\eta=0}$ are developed. A method is proposed to predict accurately the heat transfer of liquid laminar free convection by means of numerical solution of the temperature gradient.

1. INTRODUCTION

SCHMIDT and Beckmann [1] were first to make the experimental measurement of the velocity field for air free convection along an isothermal flat plate. The results have been compared with the numerical solutions based on Boussinesq's approximation. However, the Boussinesq approximation holds true only for the case of very small temperature difference $(t_w - t_\infty)$ [2]. This conclusion has been further demonstrated in our recent studies [3, 4] for gas laminar free convection in consideration of variable thermophysical properties, and were checked pretty well by new measuring results for the velocity fields of air free convection along an isothermal vertical flat plate with different temperature conditions [5].

The theoretical analysis of laminar free convection of liquid along an isothermal vertical flat plate was also started by means of the Boussinesq approximation. For the case of larger temperature difference, the effects of variable thermophysical properties should be taken into consideration, as those discussed in refs. [6–10]. The results reported so far, are not convenient for heat transfer prediction due to difficulty of treating the variable thermophysical properties in governing equations.

The present work focuses on the study of liquid laminar free convection along an isothermal vertical flat plate with consideration of variable thermophysical properties, and on the measurement of velocity fields for water laminar free convection by LDV (laser doppler velocimeter) so as to verify the theoretical prediction. We aim to extend the practicable method reported for gas laminar free convection [3, 4] to liquid, and thus clarify the character of fluid laminar free convection.

2. GOVERNING EQUATIONS

The analytical model and coordinating system for two-dimensional free convection of liquid along an isothermal vertical flat plate are shown in Fig. 1. The isothermal vertical flat plate is suspended in a quiescent liquid. The plate temperature is t_w and the ambient liquid temperature is t_∞. If $t_w \neq t_\infty$, steady two dimensional free convection will occur along the plate. We assume that the free convection is laminar. Then, the governing partial differential equations for taking into account of variable thermophysical properties will be:

$$\frac{\partial(\rho w_x)}{\partial x} + \frac{\partial(\rho w_y)}{\partial y} = 0 \quad (1)$$

$$\rho\left(w_x \frac{\partial w_x}{\partial x} + w_y \frac{\partial w_x}{\partial y}\right) = g(\rho_\infty - \rho) + \frac{\partial}{\partial y}\left(\mu \frac{\partial w_x}{\partial y}\right) \quad (2)$$

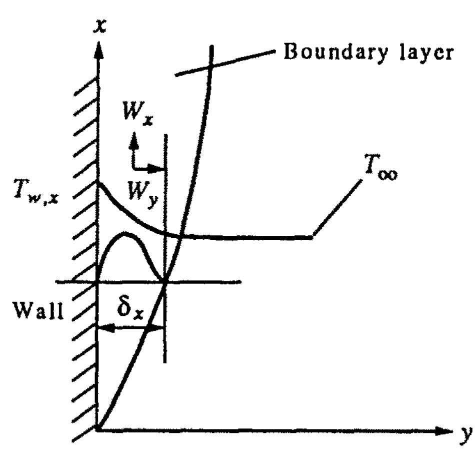

FIG. 1. Physical model coordinate system.

NOMENCLATURE

c_p specific heat at constant pressure [kJ kg^{-1} K^{-1}]
g gravitational acceleration [m s^{-2}]
$Gr_{x,\infty}$ local Grashof number
$Nu_{x,w}$ local Nusselt number
p pressure [N m^{-2}]
Pr Prandtl number, $\mu c_p/\lambda$
q_x local heat transfer rate [W m^{-2}]
t Celsius temperature [°C]
T absolute temperature [K]
w_x, w_y velocity component x and y direction respectively [m s^{-1}]
W_x, W_y dimensionless velocity component in the x- and y-direction.

Greek symbols
α_x local heat transfer coefficient [W m^{-2} K^{-1}]
δ boundary layer thickness [m]
η dimensionless coordinate variable for boundary layer
η_δ dimensionless thickness of velocity boundary layer
θ dimensionless temperature
λ thermal conductivity [W m^{-1} K^{-1}]
μ absolute viscosity [kg m^{-1} s^{-1}]
ν kinematic viscosity [m^2 s^{-1}]
ρ density [kg m^{-3}].

Subscripts
w at wall
∞ far from the wall surface.

$$c_p\left(w_x\frac{\partial t}{\partial x}+w_y\frac{\partial t}{\partial y}\right)=\frac{\partial}{\partial y}\left(\lambda\frac{\partial t}{\partial y}\right) \quad (3)$$

with boundary conditions

$$y=0:\quad w_x=0,\quad t=t_w \quad (4)$$

$$y\to\infty:\quad w_x\to 0,\quad t\to t_\infty. \quad (5)$$

Introduce the following similarity variables:

$$\eta=\frac{y}{x}\left(\tfrac{1}{4}Gr_{x,\infty}\right)^{1/4} \quad (6)$$

$$Gr_{x,\infty}=\left|\frac{g\left(\frac{\rho_\infty}{\rho_w}-1\right)x^3}{\nu_\infty^2}\right| \quad (7)$$

$$\theta=\frac{t-t_\infty}{t_w-t_\infty} \quad (8)$$

$$W_x=\left[2\sqrt{(gx)}\left|\frac{\rho_\infty}{\rho_w}-1\right|^{1/2}\right]^{-1}w_x \quad (9)$$

$$W_y=\left[2\sqrt{(gx)}\left|\frac{\rho_\infty}{\rho_w}-1\right|^{1/2}\left(\tfrac{1}{4}Gr_{x,\infty}\right)^{-1/4}\right]^{-1}w_y. \quad (10)$$

Then, equations (1)–(3) can be transformed to the following dimensionless ordinary differential equations:

$$\left(2W_x-\eta\frac{dW_x}{d\eta}+4\frac{dW_y}{d\eta}\right)-\frac{1}{\rho}\frac{d\rho}{d\eta}(-\eta W_x+4W_y)=0 \quad (11)$$

$$\frac{\nu_\infty}{\nu}\left[W_x\left(2W_x-\eta\frac{dW_x}{d\eta}\right)+4W_y\frac{dW_x}{d\eta}\right]=\frac{d^2W_x}{d\eta^2}+\frac{1}{\mu}\frac{d\mu}{d\eta}\frac{dW_x}{d\eta}+\frac{\nu_\infty}{\nu}\frac{\frac{\rho_\infty}{\rho}-1}{\frac{\rho_\infty}{\rho_w}-1} \quad (12)$$

$$Pr\frac{\nu_\infty}{\nu}(-\eta W_x+4W_y)\frac{d\theta}{d\eta}=\frac{d^2\theta}{d\eta^2}+\frac{1}{\lambda}\frac{d\lambda}{d\eta}\frac{d\theta}{d\eta} \quad (13)$$

with dimensionless boundary conditions:

$$\eta=0:\quad W_x=0,\quad W_y=0,\quad \theta=1 \quad (14)$$

$$\eta\to\infty:\quad W_x\to 0,\quad \theta\to 0. \quad (15)$$

3. TREATMENT OF VARIABLE THERMOPHYSICAL PROPERTIES

The specific heat c_p of water and many liquids change little with temperature, and so, it is possible to regard the specific heat c_p as the value $c_{p\infty}$ for engineering application. In this case, equation (13) can be transformed as follows for water and many other liquid laminar free convection:

$$Pr_\infty\frac{\rho}{\rho_\infty}\frac{\lambda_\infty}{\lambda}(-\eta W_x+4W_y)\frac{d\theta}{d\eta}=\frac{d^2\theta}{d\eta^2}+\frac{1}{\lambda}\frac{d\lambda}{d\eta}\frac{d\theta}{d\eta}. \quad (16)$$

Equations (11), (12) and (16) can be used to deal with the liquid laminar free convection, with dimensionless thermophysical property factors ρ/ρ_∞, μ_∞/μ, λ_∞/λ, ν_∞/ν, $1/\rho\, d\rho/d\eta$, $1/\mu\, d\mu/d\eta$, and $1/\lambda\, d\lambda/d\eta$ involved, and the equations can be solved numerically.

According to the typical experimental values in ref. [11], the temperature-dependent expressions of density and thermal conductivity of water with the temperature range between 0–100°C can be, respectively, obtained as:

$$\rho = -4.48\times10^{-3}t^2+999.9 \quad (17)$$

$$\lambda = -8.01\times10^{-6}t^2+1.94\times10^{-3}t+0.563. \quad (18)$$

The deviation predicted by equation (17) is less than 0.35%, and the deviation predicted by equation (18) is less than 0.18%, compared with the experimental data in ref. [11].

For absolute viscosity of water, the following expression described in ref. [12] is applied:

$$\mu = \exp\left[-1.6-\frac{1150}{T}+\left(\frac{690}{T}\right)^2\right]\times10^{-3}. \quad (19)$$

The deviation predicted by equation (19) is less than 1.8%, as compared with the experimental data in ref. [11].

Then, the thermophysical property factors in equations (11), (12) and (16) can be transformed respectively as below:

$$\frac{1}{\rho}\frac{d\rho}{d\eta} = \left[-2\times4.48\times10^{-3}t(t_w-t_\infty)\frac{d\theta}{d\eta}\right]\times(-4.48\times10^{-3}t^2+999.9)^{-1} \quad (20)$$

$$\frac{1}{\mu}\frac{d\mu}{d\eta} = \left(\frac{1150}{T^2}-2\times\frac{690^2}{T^3}\right)(t_w-t_\infty)\frac{d\theta}{d\eta} \quad (21)$$

$$\frac{1}{\lambda}\frac{d\lambda}{d\eta} = \left[-2\times8.01\times10^{-6}t+1.94\times10^{-3}(t_w-t_\infty)\frac{d\theta}{d\eta}\right]\times(-8.01\times10^{-6}t^2+1.94\times10^{-3}t+0.562)^{-1} \quad (22)$$

where

$$t = (t_w-t_\infty)\theta+t_\infty. \quad (23)$$

The values of ρ_∞, ρ_w, v_∞, λ_∞ and Pr_∞ for numerical calculation are taken directly from ref. [11], and are quoted here briefly as Table 1.

The shooting method [13] has been adopted to solve the equations numerically at different temperature conditions t_w and t_∞. The typical results for velocity and temperature fields are plotted as Figs. 2, 3, 8 and 9, respectively.

4. MEASUREMENTS ON VELOCITY FIELD

The experimental set-up, shown in Fig. 4, consists mainly of an isothermal vertical flat plate, a LDV and a water tank.

The isothermal vertical flat plate is made of copper, 250 mm in length, 140 mm in width and 7 mm in thickness. The plate surface is well polished, and nickel–chromium wire with 0.5 mm in diameter and 389 m in length is uniformly inserted into the plate to serve as an electric heater. The bottom of the plate is sharpened, so that the free convection velocity field to be measured would not be disturbed by the leading edge. The thermocouples are installed in the plate, very close to the measured surface. On the top of the plate, two metal plates 150 mm in length and 3 mm in thickness are welded and drilled to suspend the plate.

LDV was used to measure the velocity field of water free convection along the plate. In order to measure very small velocity such as the velocity field of water free convection, the technique of frequency deviation shift is applied to the LDV.

The water tank is made of organic glass plate 8 mm thick and is 1.1 m in length, 0.7 m in width and 0.35 m in height with top opened. One side of the tank is drilled four drill ways, 20 mm in diameter, to serve as the laser paths. The distance between each two near centres of the drilled ways is 50 mm, which just matches the measured heights. The places of the drilled ways are covered with thin organic glasses of 1 mm thick.

The measurements are carried out in two temperature conditions: $t_w = 40$ and 60°C, with t_∞ kept at 20°C. For each temperature condition, the measurements are made at four heights from the bottom of the plate, that is, $x = 0.05$, 0.10, 0.15 and 0.2 m. The typical measured values of the velocity component w_x are summarized in Figs. 5 and 6 respectively, and the values in Fig. 5 are further correlated to the dimensionless plot in Fig. 7. These data agree pretty well with the corresponding predicted values listed in Tables 2 and 3.

5. HEAT TRANSFER ANALYSIS WITH DISCUSSIONS

The local heat transfer rate per unit area from the surface to the ambient fluid can be calculated by Fourier's law as

Table 1. The values of thermophysical properties of water

t [°C]	0	20	40	60	80	100
ρ [kg m^{-3}]	999.8	998.3	992.3	983.2	971.4	958.1
v [10^{-6} m^2 s^{-1}]	1.792	1.004	0.658	0.475	0.365	0.294
λ [W m^{-1} K^{-1}]	0.562	0.5996	0.6287	0.6507	0.6668	0.677
Pr	13.44	6.99	4.34	3.00	2.23	1.76

Table 2. The typical solutions of velocities w_x for water laminar free convection along vertical plate surface with $t_w = 40°C$ and $t_\infty = 20°C$

$x = 0.05$ m				$x = 0.10$ m			
η	y [mm]	W_x	w_x	η	y [mm]	W_x	w_x
0	0	0	0	0	0	0	0
0.075	0.102	0.0378	0.0041	0.075	0.121	0.0378	0.0058
0.150	0.204	0.0674	0.0073	0.150	0.242	0.0674	0.0104
0.300	0.408	0.1063	0.0116	0.300	0.485	0.1063	0.0164
0.450	0.612	0.1253	0.0136	0.450	0.727	0.1253	0.0193
0.600	0.815	0.1314	0.0143	0.600	0.970	0.1314	0.0202
0.800	1.087	0.1277	0.0139	0.800	1.293	0.1277	0.0197
1.200	1.631	0.1046	0.0114	1.200	1.939	0.1046	0.0161
1.500	2.039	0.0853	0.0093	1.500	2.424	0.0853	0.0131
1.800	2.446	0.0681	0.0074	1.800	2.909	0.0681	0.0105
2.100	2.854	0.0536	0.0058	2.100	3.394	0.0536	0.0083

$x = 0.15$ m				$x = 0.20$ m			
η	y [mm]	W_x	w_x	η	y [mm]	W_x	w_x
0	0	0	0	0	0	0	0
0.075	0.134	0.0378	0.0071	0.075	0.144	0.0378	0.0082
0.150	0.268	0.0674	0.0127	0.150	0.288	0.0674	0.0147
0.300	0.537	0.1063	0.0201	0.300	0.577	0.1063	0.0232
0.450	0.805	0.1253	0.0236	0.450	0.865	0.1253	0.0273
0.600	1.073	0.1314	0.0248	0.600	1.153	0.1314	0.0286
0.800	1.431	0.1277	0.0241	0.800	1.538	0.1277	0.0278
1.200	2.147	0.1046	0.0197	1.200	2.306	0.1046	0.0228
1.500	2.684	0.0853	0.0161	1.500	2.883	0.0853	0.0186
1.800	3.220	0.0681	0.0129	1.800	3.460	0.0681	0.0148
2.100	3.757	0.0536	0.0101	2.100	4.036	0.0536	0.0117

Table 3. The typical solutions of velocities w_x for water laminar free convection along vertical plate surface with $t_w = 60°C$ and $t_\infty = 20°C$

$x = 0.05$ m				$x = 0.10$ m			
η	y [mm]	W_x	w_x	η	y [mm]	W_x	w_x
0	0	0	0	0	0	0	0
0.075	0.081	0.0454	0.0079	0.075	0.096	0.0454	0.0111
0.150	0.162	0.0789	0.0137	0.150	0.192	0.0789	0.0194
0.300	0.323	0.1189	0.0206	0.300	0.384	0.1189	0.0292
0.450	0.485	0.1345	0.0233	0.450	0.576	0.1345	0.0330
0.600	0.646	0.1360	0.0236	0.600	0.768	0.1360	0.0334
0.800	0.862	0.1273	0.0221	0.800	1.024	0.1273	0.0313
1.200	1.292	0.1001	0.0174	1.200	1.536	0.1001	0.0246
1.650	1.777	0.0721	0.0125	1.650	2.112	0.0721	0.0177
2.250	2.423	0.0446	0.0077	2.250	2.880	0.0446	0.0109

$x = 0.15$ m				$x = 0.20$ m			
η	y [mm]	W_x	w_x	η	y [mm]	W_x	w_x
0	0	0	0	0	0	0	0
0.075	0.106	0.0454	0.0137	0.075	0.114	0.0454	0.0158
0.150	0.213	0.0789	0.0237	0.150	0.228	0.0789	0.0274
0.300	0.425	0.1189	0.0358	0.300	0.457	0.1189	0.0413
0.450	0.638	0.1345	0.0404	0.450	0.685	0.1345	0.0467
0.600	0.850	0.1360	0.0409	0.600	0.913	0.1360	0.0472
0.800	1.134	0.1273	0.0383	0.800	1.218	0.1273	0.0442
1.200	1.700	0.1001	0.0301	1.200	1.826	0.1001	0.0348
1.650	2.338	0.0721	0.0217	1.650	2.511	0.0721	0.0250
2.250	3.188	0.0446	0.0134	2.250	3.425	0.0446	0.0155

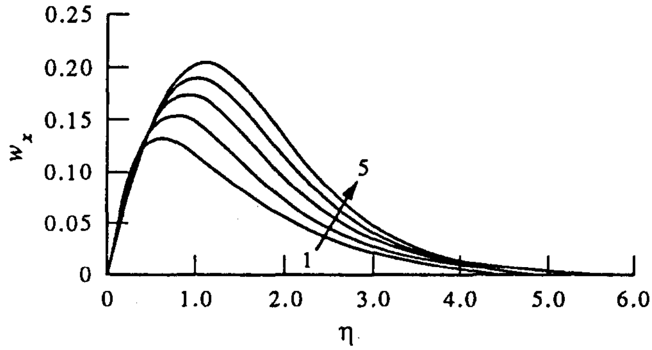

FIG. 2. The velocity profiles at different t_∞ for $t_w = 40°C$. ($1 \rightarrow 5: t_\infty = 20, 39.99, 60, 80, 100°C$.)

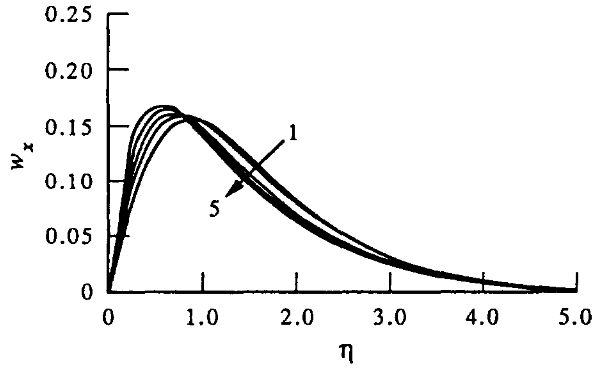

FIG. 3. The velocity profiles at different surface temperature t_w, for water laminar free convection with $t_\infty = 40°C$. ($1 \rightarrow 5: t_w = 20, 39.99, 60, 80, 100°C$.)

$$q_x = -\lambda_w \frac{\partial t}{\partial y}\bigg|_{y=0} \quad \text{or} \quad q_x = -\lambda_w \left(\frac{\partial t}{\partial \eta}\frac{\partial \eta}{\partial y}\right)\bigg|_{y=0}$$

and therefore, from equation (6),

$$q_x = -\lambda_w (t_w - t_\infty)\left(\frac{1}{4}Gr_{x,\infty}\right)^{1/4} x^{-1} \frac{d\theta}{d\eta}\bigg|_{\eta=0}. \quad (24)$$

The local heat transfer coefficient can be expressed by

$$\alpha_x = -\lambda_w \left(\frac{1}{4}Gr_{x,\infty}\right)^{1/4} x^{-1} \frac{d\theta}{d\eta}\bigg|_{\eta=0}. \quad (25)$$

The local Nusselt number, defined as $Nu_{x,w} = \alpha_x x/\lambda_w$, will be

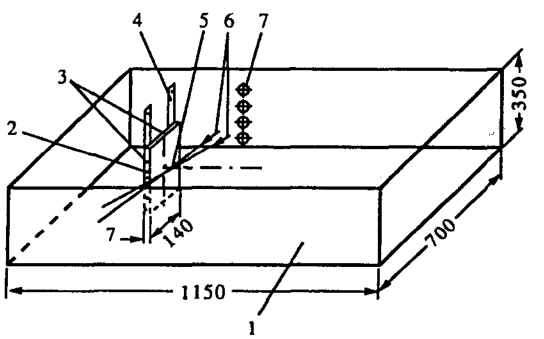

FIG. 4. Schematic diagram of the experiment set-up: 1—water tank, 2—isothermal vertical flat plate, 3—thermocouples, 4—metal plates, 5—laser focus, 6—laser paths, 7—drilled ways.

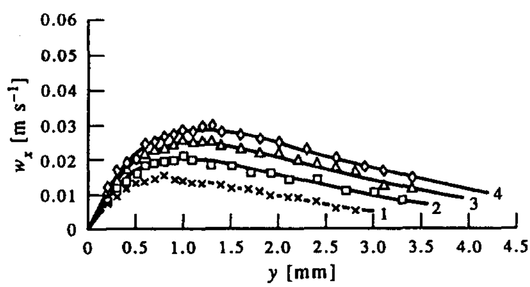

FIG. 5. Measured and calculated values for velocity of water laminar free convection with $t_w = 40°C$ and $t_\infty = 20°C$. (Full line: numerical solution, symbol: corresponding measured value 1. $x = 0.05$ m, 2. $x = 0.10$ m, 3. $x = 0.15$ m, 4. $x = 0.20$ m.)

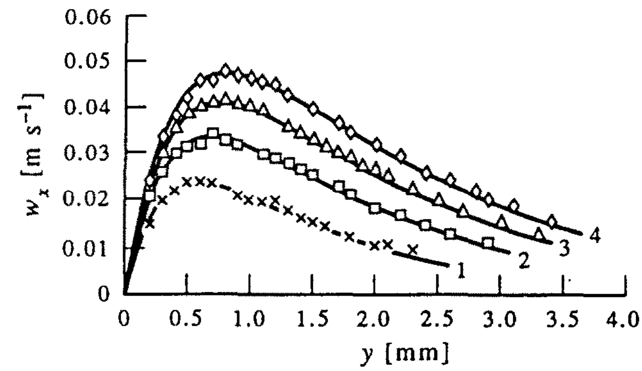

FIG. 6. Measured and calculated values for velocity of water laminar free convection with $t_w = 60°C$ and $t_\infty = 20°C$. (Full line: numerical solution, symbol: corresponding measured value 1. $x = 0.05$ m, 2. $x = 0.10$ m, 3. $x = 0.15$ m, 4. $x = 0.20$ m.)

$$Nu_{x,w} = -\left(\frac{1}{4}Gr_{x,\infty}\right)^{1/4} \left.\frac{d\theta}{d\eta}\right|_{\eta=0}. \tag{26}$$

It is obvious that the velocity and temperature fields for water laminar free convection can be solved from the governing ordinary differential equations (11), (12) and (16) with the boundary conditions, equations (14) and (15), combined with equations (17)–(23), and so we can obtain the $Nu_{x,\infty}$ value from equation (26). It is expected that the dimensionless velocity field W_x and W_y, and the dimensionless temperature field will be functions of temperature conditions t_w and t_∞.

By the Shooting Method [13], the non-linear governing equations (11), (12) and (16) with the boundary conditions (14) and (15) are solved numerically at different temperature conditions t_w and t_∞. The property values ρ, μ, λ, ν and Pr of water at different temperature are taken from ref. [11], as quoted in Table 1. The typical numerical solutions for the temperature gradient $-(d\theta/d\eta)|_{\eta=0}$ are described in Table 4 and shown in Fig. 10.

The case $t_w \to t_\infty$ is corresponding to the Boussinesq approximation. From Table 4, the expression of the numerical solution $-(d\theta/d\eta)|_{\eta=0}$ for Boussinesq approximation, i.e. for the case of constant properties, can be described as follows:

$$-\left.\frac{d\theta}{d\eta}\right|_{\eta=0} = 0.5764 + 0.1797 \ln Pr_\infty + 0.0331 \ln^2 Pr_\infty. \tag{27}$$

The predicted deviation by equation (27) is less than 0.79%.

From Table 1, as the temperature t_∞ increases from 0 to 100°C, the value of Pr_∞ for water will decrease from 13.44 to 1.76. It can be seen from Fig. 10 that the effect of Pr_∞ on the solution $(-(d\theta/d\eta)|_{\eta=0})$ is very obvious, but the effect of t_w on the solution $(-(d\theta/d\eta)|_{\eta=0})$ is small in general, especially for higher value t_∞, i.e. for smaller Pr_∞. If equation (27) is taken for prediction of the numerical solution $(-(d\theta/d\eta)|_{\eta=0})$ for different wall temperature t_w, the maximal deviation is less than 6% for t_∞ ranging from 5 to 100°C, and will be less than 2% for t_∞ ranging from 50 to 100°C, as shown in Fig. 11.

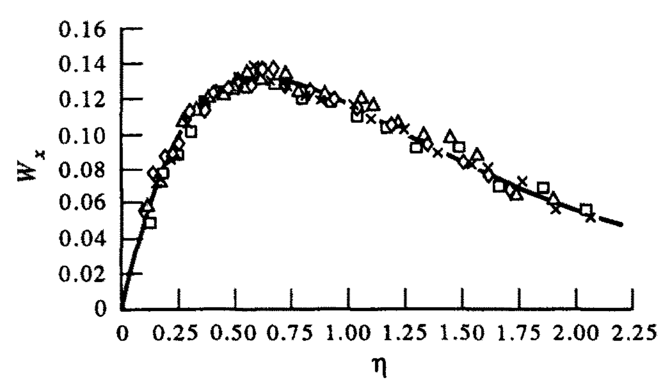

FIG. 7. Measured and calculated values for dimensionless velocity distribution of water laminar free convection with $t_w = 40°C$ and $t_\infty = 20°C$. (Full line: numerical solution, symbol: corresponding measured value 1. $x = 0.05$ m, 2. $x = 0.10$ m, 3. $x = 0.15$ m, 4. $x = 0.20$ m.)

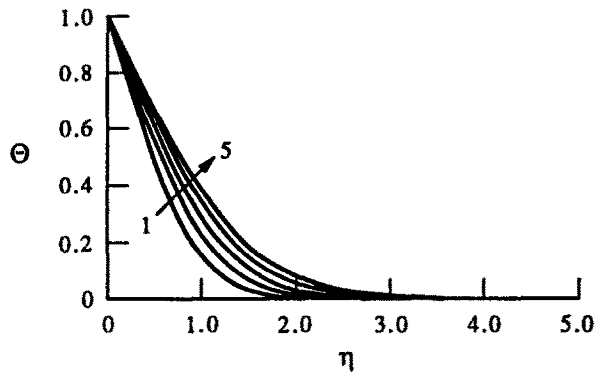

FIG. 8. The temperature profiles at different t_∞, for water laminar free convection along vertical plate surface with $t_w = 40°C$. (1 → 5: t_∞ = 20, 39.99, 60, 80, 100°C.)

6. CONCLUSION

Investigations have been carried out to study the effect of variable thermophysical properties on water laminar free convection along an isothermal vertical plate surface. The temperature-dependent expressions of the density, thermal conductivity and viscosity are involved, while the specific heat at constant pressure is taken as constant with maximum possible deviation of 0.45% only.

Table 4. The typical numerical solutions of dimensionless temperature gradient $-(d\theta/d\eta)|_{\eta=0}$ for water laminar free convection along a vertical flat plate

t_w	t_∞	Pr_∞	$-(d\theta/d\eta)\|_{\eta=0}$	t_w	t_∞	Pr_∞	$-(d\theta/d\eta)\|_{\eta=0}$
4.99	5	11.16	1.210	5	10	9.42	1.153
10	5	11.16	1.169	9.99	10	9.42	1.137
15	5	11.16	1.158	15	10	9.42	1.133
20	5	11.16	1.156	20	10	9.42	1.130
30	5	11.16	1.164	30	10	9.42	1.131
40	5	11.16	1.179	40	10	9.42	1.139
50	5	11.16	1.196	50	10	9.42	1.150
60	5	11.16	1.212	60	10	9.42	1.162
70	5	11.16	1.229	70	10	9.42	1.175
80	5	11.16	1.245	80	10	9.42	1.187
90	5	11.16	1.260	90	10	9.42	1.199
100	5	11.16	1.275	100	10	9.42	1.211
5	20	6.99	1.076	5	30	5.42	0.989
10	20	6.99	1.063	10	30	5.42	0.983
15	20	6.99	1.056	15	30	5.42	0.979
19.99	20	6.99	1.050	20	30	5.42	0.977
30	20	6.99	1.051	29.99	30	5.42	0.971
40	20	6.99	1.054	40	30	5.42	0.977
50	20	6.99	1.060	50	30	5.42	0.983
60	20	6.99	1.068	60	30	5.42	0.988
70	20	6.99	1.075	70	30	5.42	0.994
80	20	6.99	1.083	80	30	5.42	0.999
90	20	6.99	1.092	90	30	5.42	1.005
100	20	6.99	1.100	100	30	6.42	1.012
5	40	4.34	0.917	5	50	3.57	0.858
10	40	4.34	0.913	10	50	3.57	0.856
15	40	4.34	0.911	15	50	3.57	0.855
20	40	4.34	0.910	20	50	3.57	0.855
30	40	4.34	0.910	30	50	3.57	0.856
39.99	40	4.34	0.914	40	50	3.57	0.858
50	40	4.34	0.916	49.99	50	3.57	0.861
60	40	4.34	0.920	60	50	3.57	0.864
70	40	4.34	0.924	70	50	3.57	0.867
80	40	4.34	0.929	80	50	3.57	0.870
90	40	4.34	0.933	90	50	3.57	0.874
100	40	4.34	0.938	100	50	3.57	0.878

Table 4. (*Continued*)

t_w	t_∞	Pr_∞	$-(d\theta/d\eta)\|_{\eta=0}$	t_w	t_∞	Pr_∞	$-(d\theta/d\eta)\|_{\eta=0}$
5	60	3	0.809	5	70	2.57	0.768
10	60	3	0.808	10	70	2.57	0.767
15	60	3	0.807	15	70	2.57	0.767
20	60	3	0.807	20	70	2.57	0.767
30	60	3	0.809	30	70	2.57	0.769
40	60	3	0.810	40	70	2.57	0.770
50	60	3	0.813	50	70	2.57	0.772
59.99	60	3	0.814	60	70	2.57	0.774
70	60	3	0.818	69.99	70	2.57	0.779
80	60	3	0.821	80	70	2.57	0.779
90	60	3	0.824	90	70	2.57	0.781
100	60	3	0.827	100	70	2.57	0.783
5	80	2.23	0.733	5	90	1.97	0.704
10	80	2.23	0.732	10	90	1.97	0.704
15	80	2.23	0.733	15	90	1.97	0.704
20	80	2.23	0.733	20	90	1.97	0.704
30	80	2.23	0.734	30	90	1.97	0.705
40	80	2.23	0.735	40	90	1.97	0.706
50	80	2.23	0.737	50	90	1.97	0.707
60	80	2.23	0.739	60	90	1.97	1.709
70	80	2.23	0.740	70	90	1.97	0.710
79.99	80	2.23	0.740	80	90	1.97	0.711
90	80	2.23	0.744	89.99	90	1.97	0.714
100	80	2.23	0.746	100	90	1.97	0.714
5	100	1.76	0.679				
10	100	1.76	0.679				
15	100	1.76	0.679				
20	100	1.76	0.679				
30	100	1.76	0.680				
40	100	1.76	0.681				
50	100	1.76	0.682				
60	100	1.76	0.683				
70	100	1.76	0.684				
80	100	1.76	0.685				
90	100	1.76	0.686				
99.99	100	1.76	0.686				

The non-linear governing equations with corresponding boundary conditions are solved numerically by the Shooting Method, and the predicted velocity fields at different temperature conditions checked well by measuring results, as shown in Figs. 5–7. This makes clear that the method proposed, to predict the heat transfer for liquid laminar free convection in coordination of variable thermophysical properties, would be reliable. The reported expression, equation (27), could be used to predict $Nu_{x,\infty}$ approximately for water laminar free convection along an isothermal vertical plate surface with the maximum deviation of less than 6% for t_∞ ranging from 5 to 100°C, and less than 2% for temperature t_∞ ranging from 50–100°C, respectively. This method will be convenient for treating and can yield accurate results for engineering applications.

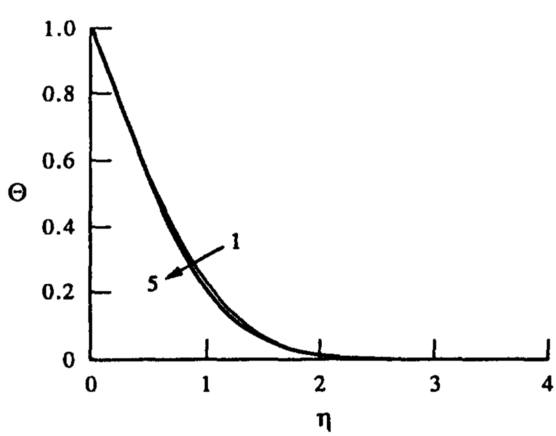

FIG. 9. The temperature profiles at different t_w, for $t_\infty = 40$°C. ($1 \rightarrow 5$: $t_w = 20, 39.99, 60, 80, 100$°C.)

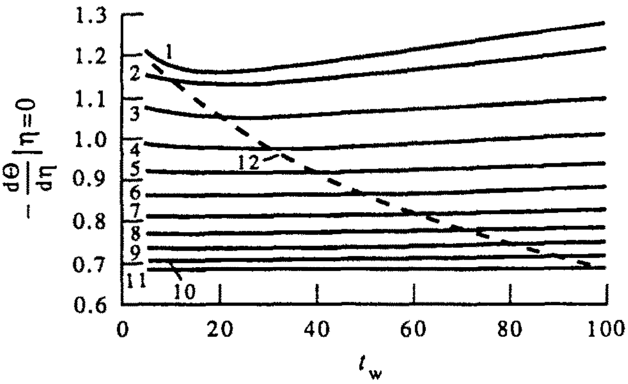

FIG. 10. Dimensionless temperature gradient for water laminar free convection along vertical plate surface. (1—$t_\infty = 5°C$, 2—$t_\infty = 10°C$, 3—$t_\infty = 20°C$, 4—$t_\infty = 30°C$, 5—$t_\infty = 40°C$, 6—$t_\infty = 50°C$, 7—$t_\infty = 60°C$, 8—$t_\infty = 70°C$, 9—$t_\infty = 80°C$, 10—$t_\infty = 90°C$, 11—$t_\infty = 100°C$, 12—solution for Boussinesq approximation.)

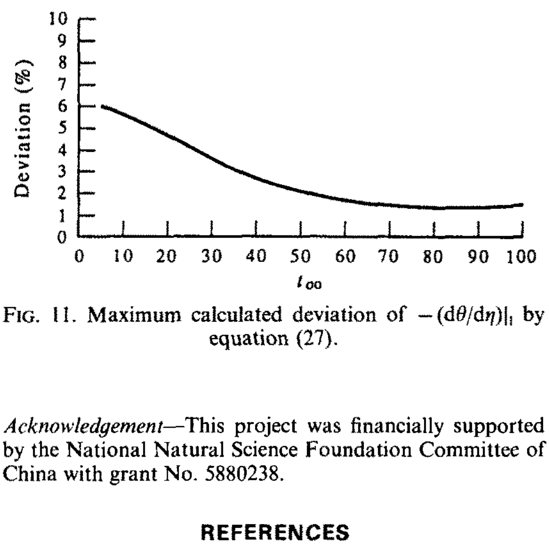

FIG. 11. Maximum calculated deviation of $-(d\theta/d\eta)|_1$ by equation (27).

Acknowledgement—This project was financially supported by the National Natural Science Foundation Committee of China with grant No. 5880238.

REFERENCES

1. E. Schmidt and W. Beckmann, Das Temperatur- und Geschwindigkeitsfeld vor einer Wärme abgebenden senkrechten Platte bei naturlicher Konvektion, *Forsch. Ing.-Wes.* **1**, 391 (1930).
2. D. D. Gray and A. Giorgini, The validity of Boussinesq approximations for liquids and gases, *Int. J. Heat Mass Transfer* **19**, 545–551 (1976).
3. D. Y. Shang and B. X. Wang, Effect of variable thermophysical properties on laminar free convection of gas, *Int. J. Heat Mass Transfer* **33**, 1387–1395 (1990).
4. D. Y. Shang and B. X. Wang, Effect of variable thermophysical properties on laminar free convection of polyatomic gas, *Int. J. Heat Mass Transfer* **34**, 749–775 (1991).
5. D. Y. Shang and B. X. Wang, Measurement on velocity field of laminar boundary layer for gas free convection along an isothermal vertical flat plate, *Heat Transfer, 3rd UK National Conference Incorporating 1st Eurotherm Conference on Thermal Sciences*, Vol. I, pp. 483–489. I.Ch.mE.(U.K.) and Hemisphere, London (1992).
6. S. Akagi, Free convection heat transfer in viscous oil, *Proceedings of Japanese Society of Mechanical Engineers* **30**(213), 624–632 (1964).
7. J. M. Piau, Influence des variations des properties physiques et la stratification en convection naturelle, *Int. J. Heat Mass Transfer* **17**, 465–476 (1974).
8. V. P. Carey and J. C. Mollendorf, Natural convection in liquids with temperature dependence viscosity. In *Proceedings of the Sixth International Heat Transfer Conference*, Toronto, NC-5, Vol. 2, pp. 211–217. Hemisphere, Washington, D.C. (1978).
9. P. Sabhapathy and K. C. Cheng, The effect of temperature-dependent viscosity and coefficient of thermal expansion on the stability of laminar, natural convective flow along an isothermal, vertical surface, *Int. J. Heat Mass Transfer* **29**, 1521–1529 (1986).
10. Z. H. Qureshi and B. Gebhart, The stability of vertical thermal buoyancy induced flow in cold pure and saline water, *Int. J. Heat Mass Transfer* **29**, 1383–1392 (1986).
11. VDI-Wärmeatlas, Berechnumgsblätter für den Wärmeübertragung, 5, erweiterte Auflage, VDI Verlage GmbH, Duesseldorf (1988).
12. J. Q. Chang, *Real Fluid Mechanics*. Tsinghua University Press, Beijing (1986).
13. I. Gladwell and D. K. Sayers, *Computational Techniques for Ordinary Differential Equations*. Academic Press, London (1980).